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DERIVATION OF EQUATIONS FOR MAXIMUM VDS TOWLINE TENSIONS UNDER --ETC(U)

AUG 64 J R SOLIN

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U. S. Navy Underwater Sound Laboratory
Fort Trumbull, New London, Connecticut

6 DERIVATION OF EQUATIONS FOR MAXIMUM
VDS TOWLINE TENSIONS UNDER DYNAMIC CONDITIONS

by

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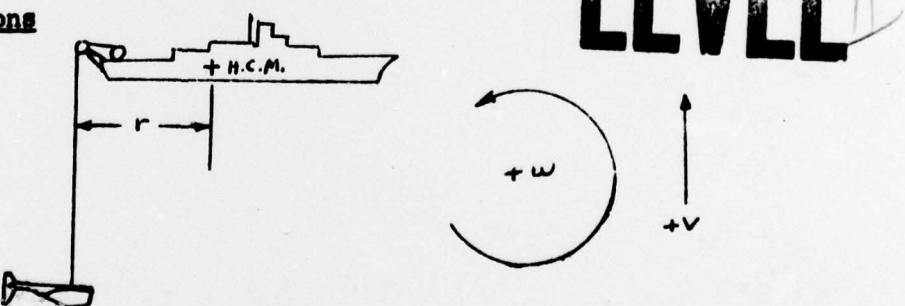
INTRODUCTION

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USL Report No. 558 (reference (a)) discusses the effects of certain ship motions on cable tensions in systems for handling submerged bodies. This memorandum derives two equations for computing VDS towline tensions under dynamic conditions. These equations differ from that shown in USL Report 558.

DERIVATION I

Terms and Notations



LEVEL II

I = hydrodynamic moment of inertia of ship about axis through its hydrodynamic center of mass (H.C.M.) and perpendicular to drawing.

M_g = hydrodynamic mass of ship in vertical direction; both I and M_g are functions of ship pitch.

M = hydrodynamic mass of fish (see equation (6) pg. 7 of Report No. 558.).

r = perpendicular distance from towable to H. C. M. (see diagram).

v = angular velocity of ship before cable tensioning.

v' = angular velocity of ship after cable tensioning.

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Terms and notations cont.

v_{3z} = vertical velocity of fish before cable tensioning.

v'_{3z} = vertical velocity of fish after cable tensioning.

v = vertical velocity of H. C. M. before cable tensioning.

v' = vertical velocity of H. C. M. after cable tensioning.

$v_{3s} = v - vr$ = vertical velocity of towpoint on ship before cable tensioning.

Also see glossary of terms in Report No. 558.

Solution

Equation 11 of USL Report No. 558 reads:

$$\Delta E_k = \frac{1}{2} M [v_{3z}^2 \pm v_{3s}^2] = \frac{T^2 l_c}{2 A_c E_c} \quad (1)$$

where T is the maximum tension a towable experiences in towing a VDS fish in heavy seas (small ship speeds assumed).

The following analysis derives a more correct expression for ΔE_k and T :

Assuming that the interaction leading to maximum cable tension is nearly instantaneous, we may neglect friction, damping, and gravitational impulses and work and say that all kinetic energy lost during the fish+ship interaction is stored as potential energy in the cable. Further we shall assume that the mass of the towable is small compared to the mass of the fish or ship, and that the fish is a point mass.

Conservation of momentum:

$$Mv_{3z} + M_s v = Mv'_{3z} + M_s v'$$

$$v'_{3z} = v_{3z} + \frac{M_s}{M} (v - v') \quad (2)$$

Conservation of angular momentum:

$$-Mrv_{3z} + Iw = -Mrv'_{3z} + Iw' \quad (3)$$

Substituting (2) into the above expression we obtain:

$$\omega' = \frac{M_s r}{I} (v - v') + \omega \quad (4)$$

Since we have assumed that the speed of the ship is zero (see pg. 3 of Report No. 558):

$$\Delta E_K = \frac{1}{2} (I \omega^2 + M v_{3z}^2 + M_s v^2) - \frac{1}{2} (I \omega'^2 + M v'_{3z}^2 + M_s v'^2) \quad (5)$$

Differentiating the above expression with respect to v' , substituting the first derivatives of (2) and (3) where specified, and setting the result equal to zero we obtain:

$$\begin{aligned} \frac{d \Delta E_K}{d v'} &= - (I \omega' \frac{d \omega'}{d v'} + M v'_{3z} \frac{d v'_{3z}}{d v'} + M_s v') = M_s (v' r + v'_{3z} - v') = 0 \\ &v'_{3z} = v' - v' r \\ \frac{d^2 \Delta E_K}{d v'^2} &\text{ is negative so } \Delta E_K \text{ is maximum when } v'_{3z} = v' - v' r \end{aligned} \quad (6)$$

Combining (2), (3), and (6) we obtain:

$$v' = \frac{(I + M r^2) M_s v + I M (v_{3z} + v r)}{I (M + M_s) + M M_s r^2} \quad (7)$$

$$w' = \frac{M M_s r (v - v_{3z}) + I \omega (M + M_s)}{I (M + M_s) + M M_s r^2} \quad (8)$$

Substituting (6), (7), and (8) into (5) we obtain:

$$(\Delta E_K)_{\max.} = \frac{\frac{1}{2} M [v_{3z} - (v - v r)]^2}{1 + \frac{M}{M_s} + \frac{M r^2}{I}} = \frac{\frac{1}{2} M [v_{3z} - v_{3s}]^2}{1 + \frac{M}{M_s} + \frac{M r^2}{I}} \quad (9)$$

ΔE_K = potential energy stored in cable = $\int_0^T T dl = \int_0^T \frac{T l_c}{A_c E_c} dT = \frac{T^2 l_c}{2 A_c E_c}$
assuming $\frac{l_c}{A_c E_c}$ = constant.

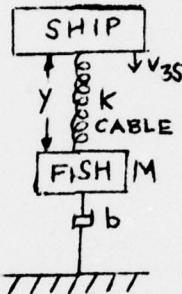
$$(\Delta E_K)_{\max.} = \frac{\frac{1}{2} M [v_{3z} - v_{3s}]^2}{1 + \frac{M}{M_s} + \frac{M r^2}{I}} = \frac{(T_{\max.})^2 l_c}{2 A_c E_c} \quad (10)$$

$$T_{\max.} = \sqrt{\frac{M A_c E_c (v_{3z} - v_{3s})^2}{1 + \frac{M}{M_s} + \frac{M r^2}{I}}} \quad (11)$$

DERIVATION II

In the previous analysis we neglected gravitational and damping forces in order to apply the conservation of momentum relations. In the following analysis gravitational and damping forces are included.

Terms and Notations



W = weight of towed body in water

b = damping coefficient

t = time

t_3 = time when body is recaptured by the towline

$t' = t - t_3$

$k_1 = 2 \pi f$

y = distance of the fish from towpoint on ship

y_0 = relaxed length of the towcable

v_{3s} = velocity of towpoint on ship at time of cable tensioning.

T = tension in the towcable $= -k (y - y_0)$ where

k = spring constant of cable $= \frac{A_c E_c}{L_c}$

Solution

In this analysis we shall assume that the motion of the ship is defined by equation (1) [$X_s = -X_0 \sin \omega t$] of Report No. 558 and therefore is unaffected by the tensioning of the cable. As we shall show later,

the time that elapses until the tension is a maximum is very small; and so we shall consider the velocity of the ship to be constant during the interaction.

$$\sum F_{\text{FISH}} = -W - K(y - y_0) - b\left(\frac{dy}{dt} + v_{ss}\right) = M \frac{d^2(y + v_{ss}t')}{dt'^2} = M \frac{dy}{dt'^2} \quad (12)$$

Under initial conditions , $t' = 0$

$$\frac{dy}{dt'} = v_{ss} - v_{ss} \quad \text{and} \quad y = y_0$$

$\therefore y =$

$$e^{-\frac{bt'}{2M}} \left[\frac{W + bv_{ss}}{K} \cos \frac{\sqrt{4MK - b^2}}{2M} t' + \frac{2MK(v_{ss} - v_{ss}) + bW + b^2v_{ss}}{K\sqrt{4MK - b^2}} \sin \frac{\sqrt{4MK - b^2}}{2M} t' \right] \quad (13)$$

$$+ \frac{Ky_0 - bv_{ss} - W}{K}$$

To find the minimum value of y we differentiate the above, set it equal to zero, and find that at y_{\min} ,

$$t' = \frac{2M}{\sqrt{4MK - b^2}} \tan^{-1} \frac{(v_{ss} - v_{ss})\sqrt{4MK - b^2}}{b(v_{ss} + v_{ss}) + 2W} \quad (14)$$

where $\tan^{-1} \frac{(v_{ss} - v_{ss})\sqrt{4MK - b^2}}{b(v_{ss} + v_{ss}) + 2W}$ is the smallest angle greater than zero.
 t' is of the order of $\sqrt{\frac{M}{K}} \approx 1$ so our original assumption is justified.

$$T_{\max} = -K(y_{\min} - y_0) =$$

$$e^{\left[\frac{-b}{\sqrt{4MK - b^2}} \tan^{-1} \frac{(v_{ss} - v_{ss})\sqrt{4MK - b^2}}{b(v_{ss} + v_{ss}) + 2W} \right]} \left[\sqrt{MK(v_{ss} - v_{ss})^2 + bW(v_{ss} + v_{ss}) + b^2v_{ss}^2 + W^2} \right] + W + bv_{ss} \quad (15)$$

In cases where $|v_{ss} - v_{ss}|$ is not small (i.e., in cases involving large tensions)

$$T_{\max} \approx \sqrt{MK(v_{ss} - v_{ss})^2} \quad (16)$$

which is nearly the same as equation (11).

COMMENTS

Comparisons with experimental data, such as those shown in reference (b), indicate that equations (11) and (16) agree more nearly with the experimental data than does equation (1).

For example:

From equation (1),

$$T = 79,000 \text{ lbs.}$$

From equation (11), and assuming that $\frac{M}{M_s} + \frac{Mr^2}{I} = 0$ $T = 36,200 \text{ lbs.}$

From equation (16), $T = 38,580 \text{ lbs.}$

$$T =$$

The experimentally measured value is 30,000 lbs. If M_s , r and I were known, the tension computed from equation (11) would be reduced and approach the value of 30,000 lbs.

CONCLUSIONS

Two analyses have been presented in this memorandum to represent the two probable extremes of an exact fish-ship mathematical model subjected to dynamic conditions.

The first analysis neglects gravitational and damping forces; as a consequence, the tension it predicts will be lower than the actual tension. Because it is based on the assumption that the interaction is nearly instantaneous, it is not accurate when $|V_{32} - V_{35}|$ is small.

The second analysis neglects the effects of ship mass; thus, it predicts a higher than actual tension.

In cases where $|V_{32} - V_{35}|$ is not small the two approaches give approximately the same tension.

Either equation (11) or (16) should be used in place of equations (11) and (11A) in USL Report No. 558.

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Student Aide

USL Tech. Memo.
No. 933-257-64

LIST OF REFERENCES

- (a) USL Report 558, "Effects of Certain Ship Motions on Cable Tensions in Systems for Handling Submerged Bodies," dtd 7 August 1962, by F. B. Rakoff
- (b) USL Tech. Memo. No. 933-121-64, "Results of Initial Experiments to Confirm Calculations of Slack Towline Phenomena," dtd 2 June 1964, by Fred J. Contrata, Jr.

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